HW#12 Solutions

7.24

(a) $\delta_s = [\pi f \mu \sigma]^{-1/2} = [\pi \times 10^7 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7]^{-1/2}$

$= 0.021\ mm$

$\longrightarrow \dfrac{d}{\delta_s} = \dfrac{0.5mm}{0.021} \sim 25$

(b)

$$R_s = \frac{1}{\sigma \delta_s} = \frac{1}{5.8 \times 10^7 \times 2.1 \times 10^{-5}} = 8.2 \times 10^{-4}\ \Omega$$

(c)

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$= \frac{8.2 \times 10^{-4}}{2\pi}\left(\frac{1}{5 \times 10^{-3}} + \frac{1}{10^{-2}}\right) = 0.039\ \Omega/m$$

7.26

$$\vec{E} = 3\cos(\pi \times 10^7 t + kx)\hat{y} - 2\cos(\pi \times 10^7 t + kx)\hat{z}$$

$$\eta \simeq \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi$$

$$S_{ave.} = \frac{-[3^2 + 2^2]}{2\eta}\hat{x} = -\frac{13}{80\pi} = -0.05\hat{x}\ W/m^2$$

7.30

$$\begin{cases} S = 1\,mW/cm^2 = 10^{-3}\,W/cm^2 = 10\,W/m^2 \\ S = \dfrac{|E(R)|^2}{2\eta_0} \end{cases}$$

$$\longrightarrow 10 = \left(\frac{3 \times 10^3}{R}\right)^2 \times \frac{1}{2 \times 120\pi} = \frac{1.2 \times 10^4}{R^2}$$

$$R = \left(\frac{1.2 \times 10^4}{10}\right)^{1/2} = 34.64$$

7.31 $\vec{E} = E_0 \cos(\omega t - ky)\hat{x}$ V/m

(a) $\longrightarrow \vec{H} = \frac{-E_0}{\eta_0} \cos(\omega t - ky)\hat{z}$

$$\vec{S}(t) = \vec{E} \times \vec{H} = \frac{E_0^2}{\eta_0} \cos^2(\omega t - ky)\hat{y}$$

$$\longrightarrow P(t) = S(t)A\Big|_{y=0} - S(t)A\Big|_{y=b} = \frac{E_0^2}{\eta_0} \underbrace{ac}_{A}\left[\cos^2\omega t - \cos^2(\omega t - kb)\right]$$

(b) $$P_{ave.} = \frac{1}{T}\int_0^T P(t)\,dt$$

$$T = \frac{2\pi}{\omega}$$

$$\longrightarrow P_{ave.} = \frac{E_0^2 ac}{\eta_0}\left\{\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[\cos^2\omega t - \cos^2(\omega t - kb)\right]dt\right\} = 0$$

this is expected since the box is in a lossless medium (air)